UNLIMITED

Tech Memo
SPACE 384

Tech Memo
SPACE 384

DEFENCE
RESEARCH
AGENCY
AEROSPACE DIVISION

Technical Memorandum

May 1991

On a Problem Arising in the Computation of Orbit Perturbations

by

R. H. Gooding

RAE, Farnborough, Hampshire

UNLIMITED

■ *DESPATCH NOTE FOR DOCUMENTS*

The attached document bears the security classification ***UNLIMITED***

To:
MR PAUL A ROBEY
DTIC
Attn:DTIC-FDAC
Cameron Station-Bldg 5
Alexandria
VA 22304 6145
USA

Your Ref: Distribution No B1387

From:
DEFENCE RESEARCH INFORMATION CENTRE
Kentigern House
65 Brown Street
Glasgow G2 8EX

Tel: 041-224 2456 **Fax:** 041-224 2470

Our Ref: 0101830

Details of Documents:

302518

Hardcopy - 2

Term of issue: On retention

Reason for issue: On the recommendation of originator

For official use only

Despatched on date shown on envelope

IMPORTANT: Please read the following carefully -

This despatch note should not be separated from the document. Classified documents must be safeguarded in accordance with the relevant security regulations. If you are in doubt about the safeguards required, ask your local Security Officer. **Should there be any problem in meeting these safeguards, then please return any unwanted reports to DRIC. A request for any such documents can be made in future as necessary.**

In the case of documents issued on loan, any request for an extension of the loan period should be sent to DRIC promptly.

All such requests must include FULL-
DRIC reference number and DRIC Accession number.

DRIC/89/2 (Revised Jun.89)

0101830

CONDITIONS OF RELEASE

302518

****

DRIC U

****

DRIC Y

Reports quoted are not necessarily available to members of the public or to commercial organisations.

DEFENCE RESEARCH AGENCY
Aerospace Division
RAE Farnborough

Technical Memorandum Space 384

Received for printing 22 May 1991

ON A PROBLEM ARISING IN THE COMPUTATION OF ORBIT PERTURBATIONS

by

R. H. Gooding

SUMMARY

In the computation of perturbations for an orbiting satellite, a problem can arise with the integration of a rate-of-change quantity, the prototype expression for which is $k \cos \theta$, where θ has a quasi-linear variation with time. The problem can be dealt with by the use of definite integrals *between* epochs rather than indefinite integrals *at* epochs.

LIST OF CONTENTS

1 INTRODUCTION AND BACKGROUND

The modelling of a satellite's orbital motion involves the integration of rate-of-change quantities that represent the effects of the relevant perturbations. These quantities may be Cartesian accelerations, in which case the integration is almost bound to be purely numerical and the general procedure (method of 'special perturbations')[1] is usually attributed to Cowell. Alternatively, the quantities can be the rates of change of a suitable set of orbital elements, in which case the integration can still be entirely numerical (variation-of-parameters method) but is more likely to be partly or wholly based on analysis (method of 'general perturbations'). The analysis inevitably requires some approximating assumptions, even when the rate-of-change quantities have very simple expressions, the most important assumption usually being that the different perturbation sources can be treated separately, coupling effects being negligible.

A common application of orbit modelling (at RAE in particular)[2] has been the use of observed perturbations in the orbits of one or more satellites to improve the knowledge of various geophysical parameters associated with the atmosphere and the gravitational field[3]. Typically the practical analysis is carried out in two stages. In the first stage (for each satellite) accumulated observations of the satellite itself over a long period of time (perhaps years) are split into groups with each group spanning a much shorter period (usually a few days); each group of observations is analysed by a computer program (such as PROP[4]) that only has to model the dominant perturbations, and in particular can ignore variations that are negligible over the short time periods involved. This analysis leads to a series of element sets, one set from each orbit determination, the whole series spanning the full period of the observational data. In the second stage attention is confined to a particular element (*eg* the inclination) from each set, and its variation, from set to set, over the full period is considered. The object is to account for this variation by allowing now for all the perturbations that are significant over the extended period, and determining values (perhaps first-time, perhaps revised) for the numerical coefficients associated with one or more of the perturbation sources - sometimes the particular perturbation sources will have been omitted from the first-stage analysis, not being 'dominant', and sometimes it will be a matter of adjusting existing values that were adequate for the first-stage analysis.

A convenient way of conducting the second-stage analysis is to remove all the known effects from the selected orbital element so that, in the absence of the additional effects associated with the perturbation source being studied, the

expected (ideal) value of the 'corrected' element would be the same at every epoch (*ie* for each of the first-stage orbit determinations). This facilitates the plotting of a graph in which, the pertinent variation of the element having now been isolated, it can easily be judged how successfully it can be modelled in the evaluation of the appropriate numerical coefficients. A particular problem often arises in this removal, or 'clearing', of known effects, and it is this problem which is the subject of the present paper.

2 THE PROBLEM

We denote by ζ the orbital element assumed to have been selected for analysis, and we consider the removal (from the series of ζ values) of 'known' perturbations typified by the rate-of-change equation

$$\dot{\zeta} = k \cos \theta . \tag{1}$$

The typical situation here is as follows: k is a slowly-changing quantity, functionally dependent on some physical parameter(s) (such as a resonant pair of harmonic coefficients from the geopotential; the moon's mass; or a tidal parameter) and also on the three most constant orbital elements of the satellite (a, e and i); θ, on the other hand, is a quantity that varies quite rapidly with time, typically being a linear combination of the other three orbital elements (Ω, ω and M) and quantities such as sidereal angle.

To identify the problem associated with equation (1), we first suppose that k is exactly constant and that the rate of change of θ is also exactly constant, equal say to λ. Then to clear ζ of the perturbations in question we might simply subtract $(k/\lambda) \sin \theta$ (an indefinite integral) from the 'observed' value of ζ available at each epoch. Inasmuch as the values of k and λ both depend on the values of the orbital elements, they could in principle be computed from the 'observed' values of the element set, of which ζ itself is one. This computation would be immediate for k, but less so for λ because it involves a rate of change (*eg* is associated with the even harmonics of the geopotential).

In practice, neither k nor λ is constant, but it is only the non-constancy of λ that concerns us so we will continue to treat k as constant. It may be our hope that the variation of θ is sufficiently close to linear, and the size of the perturbation (via the value of k) sufficiently small that we can escape from complicated analysis, or more probably a full-blooded numerical integration, by just evaluating $\dot{\theta}$ (the local value of λ) at each epoch and

continuing to 'correct' each ζ by subtracting $(k/\dot{\theta}) \sin \theta$ from it. If we trust the validity of this approach, the question then arises as to how we should calculate $\dot{\theta}$ at each epoch. In some circumstances (such as the above-mentioned geopotential-defined $\dot{\omega}$) a value can be computed straightforwardly and with fair confidence. In other cases, where this is difficult, it may be thought preferable to use a numerical value for $\dot{\theta}$, based on the use of two element sets. Thus a value could be calculated from

$$\dot{\theta} = (\theta - \theta')/(t - t') ,$$

where t' is the epoch before (or after) the t-epoch tacitly assumed. The bias here is immediately objectionable, and the use of epochs both before and after t is not much better, being arbitrary, and still biased unless the epochs are evenly spaced. A particularly unsatisfactory aspect of this approach can be seen to arise when θ is changing very slowly, examples of this situation being associated with resonance and the J_2-induced critical inclination. The values of $(k/\dot{\theta}) \sin \theta$ will then be numerically large and the approach eventually loses all validity.

The problem may now be stated as the removal of perturbations, of the type under consideration, in such a way as to avoid all difficulty associated with small denominators, but without losing the simplicity attached to the underlying assumptions. It should be noted that the problem does *not* consist in the way in which a value might be estimated for $\dot{\theta}$; rather, it goes back to the way in which equation (1) should be integrated.

3 THE SOLUTION

In attempting to deal with equation (1), the mistake was to aim at a series of individual quantities, one at each epoch and independent of all the other epochs, as the perturbation-clearing quantities. This approach, which led to the indefinite integral $(k/\dot{\theta}) \sin \theta$, is legitimate for purely theoretical development, leading to formulae that are neat and compact, but it is not at all appropriate for the regular handling of data in practice.

Let us suppose, then, that we have the $N + 1$ epochs t_i , where $i = 0$, ..., N , and that we treat $\dot{\theta}$ as constant *between* successive epochs. Then the *definite* integral of equation (1) between epochs t_{i-1} and t_i is given by

$$I_i = k \int_{t_{i-1}}^{t_i} \cos\theta \, dt = (k/\dot{\theta})(\sin\theta_i - \sin\theta_{i-1}) ,$$

where the appropriate value of $\dot{\theta}$ is obviously given by:

$$\dot{\theta} = \frac{\theta_i - \theta_{i-1}}{t_i - t_{i-1}} .$$

So we simply write

$$I_i = k \frac{\sin\theta_i - \sin\theta_{i-1}}{\theta_i - \theta_{i-1}} (t_i - t_{i-1}) . \qquad (2)$$

The quantity to be removed from ζ at t_i can now be seen to be $\sum_1^i I_j$, if t_0 is regarded as the 'base epoch' at which no correction is made. This may appear to introduce a bias (towards t_0 as base epoch), but the use of a different base epoch would change all the correcting quantities, $\sum I$, by the same amount (independent of i). The important point is that we have eliminated zero-denominator difficulties at a stroke; indeed, equation (2) automatically leads to bounded quantities, since $|\sin\theta - \sin\varphi| \le |\theta - \varphi|$, with (in particular) $\sum_1^N I_j \le k(t_N - t_0)$.

It remains to deal with a rather obvious point, namely, that the computation of $(\sin\theta - \sin\varphi)/(\theta - \varphi)$ has to be arranged so as to avoid rounding error when θ and φ are close together. Since

$$\frac{\sin\theta - \sin\varphi}{\theta - \varphi} = \frac{\sin\delta}{\delta} \cos \tfrac{1}{2}(\theta + \varphi) ,$$

where $\delta = \tfrac{1}{2}(\theta - \varphi)$, this is a trivial matter involving the smooth transition[5] to a polynomial in δ when $|\delta|$ is less than some (possibly) computer-dependent criterion.

4 FINAL REMARKS

The procedure described is straightforward and, assuming the avoidance of rounding error just referred to, free from hazard. Some readers might think that the procedure could be improved by some sort of smoothing in particular circumstances, when there is significant 'sampling error' in the observed values of θ , for example. However, the rewards for 'improvements' of this type are likely to be very small; at worst, it is possible that time will be wasted in the derivation of corrections that are actually inferior to those obtained by the unmodified procedure.

REFERENCES

No.	Author	Title, etc
1	R.H. Gooding	On the integration and interpolation procedures in RAE programs for orbit generation. RAE Technical Memorandum Space 357 (1986)
2	R.H. Gooding	Analytical generation of satellite orbits at the RAE. RAE Technical Memorandum Space 345 (1984)
3	R.H. Gooding	Geophysical applications of orbit determination: a lecture. RAE Technical Memorandum Space 276 (1980)
4	R.H. Gooding	The evolution of the PROP6 orbit determination program, and related topics. RAE Technical Report 74164 (1974)
5	R.H. Gooding	Satellite motion in an axi-symmetric gravitational field. Part 1: Perturbations due to J_2 (second order) and J_3 . RAE Technical Report 88068 (1988)

[illegible]

[illegible] Hants, UK

6a. Sponsoring Agency [illegible]

7. Title
On a problem arising in the calculation of orbit perturbations

7a. (For Translations) Title in Foreign Language

7b. (For Conference Papers) Title, Place and Date of Conference

8. Author 1. Surname, Initials Gooding, R.H.	9a. Author 2	9b. Authors 3, 4	10. Date May 1991 | Pages 8 | Refs. 5
11. Contract Number	12. Period	13. Project	14. Other Reference Nos.

15. Distribution statement
(a) Controlled by –
(b) Special limitations (if any) –
If it is intended that a copy of this document shall be released overseas refer to RAE Leaflet No.3 to Supplement 6 of MOD Manual 4.

16. Descriptors (Keywords) (Descriptors marked * are selected from TEST)
[illegible]

17. Abstract
[illegible] satellite, a problem [illegible] the prototype [illegible] connection with [illegible] between [illegible]

RAE Form A143 (revised October 1980)